Structured Eigenvectors, Interlacing, and Matrix Completions

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Abstract

This dissertation presents results from three areas of applicable matrix analysis: structured eigenvectors, interlacing, and matrix completion problems. Although these are distinct topics, the structured eigenvector results provide connections.

It is a straightforward matrix calculation that if $\lambda$ is an eigenvalue of $A$, $x$ an associated structured eigenvector and $\alpha$ the set of positions in which $x$ has nonzero entries, then $\lambda$ is also an eigenvalue of the submatrix of $A$ that lies in the rows and columns indexed by $\alpha$. We present a converse to this statement and apply the results to interlacing and to matrix completion problems. Several corollaries are obtained that lead to results concerning the case of equality in the interlacing inequalities for Hermitian matrices, and to the problem of the relationship among eigenvalue multiplicities in various submatrices.

Classical interlacing for an Hermitian matrix $A$ may be viewed as describing how many eigenvalues of $A$ must be captured by intervals determined by eigenvalues of a principal submatrix of $A$. We generalize the classical interlacing theorems by using singular values of off-diagonal blocks of $A$ to construct extended intervals that capture a larger number of eigenvalues. The union of pairs is also discussed, and applications are mentioned.

The matrix completion results that we present include the positive semidefinite cycle completion problem for matrices with data from the complex numbers, distance matrix cycle completability conditions, the P-matrix completion problem, and the totally nonnegative completion problem. We show that the positive semidefinite cycle completion problem for matrices with complex data is a special case of a larger real positive semidefinite completion problem. In addition, we characterize those graphs for which the cycle conditions on all minimal cycles imply that a partial distance matrix has a distance matrix completion. We also prove that every combinatorially symmetric partial P-matrix has a P-matrix completion and we characterize the class of graphs for which every partial totally nonnegative matrix has a totally nonnegative completion. The structured eigenvector results are used to give a new proof of the maximum minimum eigenvalue completion problem for partial Hermitian matrices with a chordal graph.